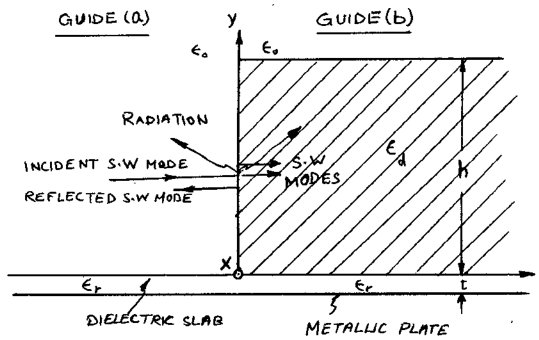

Fig. 1. Geometry of the problem treated in footnote 1.

$$(1 - R)\langle e^b(\kappa'),h_1{}^a\rangle - \int_0^\infty \Gamma(\kappa)\langle e^b(\kappa'),h^a(\kappa)\rangle\, d\kappa = T(\kappa')N^b(\kappa') \quad (2)$$

where $\langle f,g\rangle$ is an inner product defined in footnote 1 and involves an integration $w \cdot r \cdot t \cdot y \cdot N_b\,(\kappa')$ is given by

$$2\pi\beta(\kappa')V^b(\kappa')W^b(\kappa')$$

where the quantities β, V, and W are defined by [footnote 1, eqs. (1)–(3)]. The inner product $\langle e^a(\kappa),h^b(\kappa')\rangle$ is given explicitly by [footnote 1, eq. (18)]. Here we need only to write it in the form

$$\begin{aligned}\langle e^a(\kappa),h^b(\kappa')\rangle &= C_1(\kappa')\delta(\kappa - \kappa') \\ &\quad + (1 - \kappa^2)^{1/2}C_2(\kappa,\kappa')/(\kappa - \kappa') + f_1(\kappa,\kappa'). \quad (3)\end{aligned}$$

Similarly,

$$\begin{aligned}\langle e^b(\kappa'),h^a(\kappa)\rangle &= C_1(\kappa')\delta(\kappa - \kappa') \\ &\quad + (1 - \kappa'^2)^{1/2}C_2(\kappa,\kappa')/(\kappa - \kappa') + f_2(\kappa,\kappa') \quad (4)\end{aligned}$$

where $\delta(\cdot)$ is the Kronecker δ-function, $C_1(\kappa')$ and $C_2(\kappa,\kappa')$ are given by

$$C_1(\kappa') = 2\pi(1 - \kappa'^2)^{1/2}[V^a(\kappa')W^b(\kappa') + W^a(\kappa')V^b(\kappa')]$$
$$C_2(\kappa,\kappa') = -i[V^a(\kappa)W^b(\kappa') - W^a(\kappa)V^b(\kappa')]$$

and $f_{1,2}(\kappa,\kappa')$ are finite functions which can be obtained from [footnote 1, eq. (18)]. Now we substitute from (3) and (4) in (1) and (2) and integrate the singular terms separately to obtain

$$(1 + R)\langle e_1{}^a,h^b(\kappa')\rangle + C(\kappa')\Gamma(\kappa') + \int_0^\infty \Gamma(\kappa)F_1(\kappa,\kappa')\, d\kappa = T(\kappa')N^b(\kappa') \quad (5)$$

$$(1 - R)\langle e^b(\kappa'),h_1{}^a\rangle - C(\kappa')\Gamma(\kappa') - \int_0^\infty \Gamma(\kappa)F_2(\kappa,\kappa')\, d\kappa = T(\kappa')N^b(\kappa') \quad (6)$$

where

$$C(\kappa') = C_1(\kappa') + 2\pi i R_s(\kappa')$$

and $R_s(\kappa')\Gamma(\kappa')$ is the residue resulting from the integration of the term including $1/(\kappa - \kappa')$ around the singularity $\kappa = \kappa'$ after the appropriate modification of the contour of integration. The functions $F_{1,2}(\kappa,\kappa')$ represent the terms in (3) and (4) other than the singularities; i.e., they are finite for all values of κ and κ'. Furthermore, $F_{1,2}(\kappa,\kappa')$ behave as κ^{-1} as $\kappa \to \infty$ with κ' finite as can be proved from [footnote 1, eq. (18)]. From the radiation condition it can be easily shown that $\Gamma(\kappa)$ tapers off as $\kappa^{-1/2}$ or faster as $\kappa \to \infty$. Hence, the integrals in (5) and (6) are finite. The first terms on the LHS of these equations are also finite by virtue of the exponential decay of the surface-wave fields. Hence (5) and (6) can be cast in the forms

$$N^b(\kappa')T(\kappa') = C(\kappa')\Gamma(\kappa') + \text{a finite quantity } X_1(\kappa')$$
$$N^b(\kappa')T(\kappa') = -C(\kappa')\Gamma(\kappa') + \text{a finite quantity } X_2(\kappa').$$

By adding and subtracting these two equations, we conclude that both $T(\kappa')$ and $\Gamma(\kappa')$ are finite[4] quantities for all finite values of κ'. We should note, however, that at $\kappa' = 1$, (6) becomes identically zero and no definite conclusion can be made about the finiteness of $\Gamma(1)$ and $T(1)$. We may then resort to the radiation condition which necessitates the finiteness of these quantities.

The previous discussion, we believe, constitutes a proof of the absence of any singularity in the continuous spectra of the fields on both sides of the discontinuity, and hence the eligibility of expanding $\Gamma(\kappa)$ and $T(\kappa)$ in terms of Laguerre polynomials as used in footnote 1.

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[4] In [2] Rulf and Kedem adopted a fixed-order approximation for $T(\kappa)$ and $\Gamma(\kappa)$ in terms of a small parameter of discontinuity. We believe that the resulting singularities in their solution should be removed by taking a better approximation.

Comments on "Asymmetric Coupled Transmission Lines in an Inhomogeneous Medium"

EUGENIO COSTAMAGNA AND UGO MALTESE

In the above paper,[1] the terminal characteristic parameters for a uniform coupled-line four-port for the general case of an asymmetric, inhomogeneous system are derived, and some of the equivalent circuits are presented.

We have read with interest the paper, in particular the discussions on the behavior of the modes of the structure.

An alternative method for expressing the propagation constants and the terminal parameters is the use of the capacitances in air and in inhomogeneous dielectric for odd and even excitation. This is very useful in the analysis and optimization of distributed networks. Formulas and procedures have been given in [1] with equivalent circuits for comb line and interdigital sections.

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